Large Deflection, Large Amplitude Vibrations and Random Response of Symmetrically Laminated Plates

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Abstract

AN analytical solution is presented for determining large-deflection static bending, large-amplitude free and forced vibrations, and large-amplitude random response of a clamped, symmetrically laminated, rectangular, thin plate subjected to a uniformly distributed transverse loading. Both movable and immovable inplane boundary conditions are considered.

Contents

The need to understand the large deflection, large-amplitude vibration and random response of aircraft composite structures has become increasingly important as a result of military and commercial demands on current and future airplane designs. Nonlinear effects of plates have been examined by several investigators. Eisley[1] and Srinivasan[2] studied the isotropic case of nonlinear vibration of beams and plates. In recent years, the use of laminated composite plates has increased considerably. This arises from the fact that, by taking advantage of composite material properties (lightweight, high strength), the material can be used very efficiently.[3-5] For the high jet engine noise levels, the nonlinear random response has been shown to be significant.[6,7] Wentz, et al.[8] considered the nonlinear random response for the special case of A_{16}, A_{26}, D_{16}, and D_{26} all being zero (specially orthotropic) for a symmetric laminated composite plate.

The solutions presented in the present investigation are for the static, free and forced vibrational, and random responses for symmetrically laminated (generally orthotropic) rectangular plates with either inplane movable or immovable clamped boundary conditions. The nonlinear equations of motion for a symmetric, generally orthotropic laminate were derived in terms of a stress function F, and a lateral displacement W. A deflection function that represents the first mode was assumed; and corresponding to the assumed mode, a stress function satisfying the different inplane edge boundary conditions was obtained by solving the compatibility equation. Next, Galerkin's method was applied to the governing equation of motion using the assumed displacement function W, to obtain a nonlinear, differential equation of the response in time. Using the differential equation, the time-dependent terms were dropped, and the applied loading assumed constant and uniform which resulted in a nonlinear equation for the static response. By retaining the time-dependent response terms and assuming a harmonic forcing function, the nonlinear differential equation was solved using a perturbation method to yield the forced vibrational response. The free vibration was studied by allowing the amplitude of the forcing function to tend to zero in the forced response solution. The nonlinear, damped, random response at various acoustic loadings was evaluated by assuming that the excitation was stationary Gaussian, ergodic, and zero mean and by employing the equivalent linearization method.[8,9]

Using the Kirchhoff hypothesis of classical thin-plate theory, the governing equation was derived for a symmetric, generally orthotropic, laminated, composite plate in terms of the Airy's stress function F, and the out of plane direction W

$$\rho h \ddot{W} + L_1(W) - \phi(F, W) - p(t) = 0 \tag{1}$$

where

$$L_1(\ ) = D_{11}\frac{\partial^4(\ )}{\partial x^4} + 4D_{16}\frac{\partial^4(\ )}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4(\ )}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4(\ )}{\partial x \partial y^3} + D_{22}\frac{\partial^4(\ )}{\partial y^4} \tag{2}$$

and

$$\phi(V_1, V_2) = \frac{\partial^2 V_1}{\partial y^2}\frac{\partial^2 V_2}{\partial x^2} + \frac{\partial^2 V_1}{\partial x^2}\frac{\partial^2 V_2}{\partial y^2} - 2\frac{\partial^2 V_1}{\partial x \partial y}\frac{\partial^2 V_2}{\partial x \partial y} \tag{3}$$

and the compatibility equation, expressed in terms of F and W, becomes

$$L_2(F) + \frac{1}{2}\phi(W, W) = 0 \tag{4}$$

where

$$L_2(\ ) = A^*_{22}\frac{\partial^4(\ )}{\partial x^4} - 2A^*_{26}\frac{\partial^4(\ )}{\partial x^3 \partial y} + (2A^*_{12} + A^*_{66})\frac{\partial^4(\ )}{\partial x^2 \partial y^2} - 2A^*_{16}\frac{\partial^4(\ )}{\partial x \partial y^3} + A^*_{11}\frac{\partial^4(\ )}{\partial y^4} \tag{5}$$

The A^*_{ij}'s and D_{ij}'s are laminate stiffnesses, h is the total laminate thickness, ρ is the mass density, E_2 is the inplane transverse lamina modulus, and $P(t)$ is the loading.

For a clamped, rectangular, symmetrically laminated composite plate of dimensions a, b with the origin located at the center of the plate, a deflection function which satisfies the boundary conditions on all four edges of the plate is

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$$W=\frac{q(t)h}{4}\left(1+\cos\frac{2\pi x}{a}\right)\left(1+\cos\frac{2\pi y}{b}\right) \tag{6}$$

Using the assumed displacement function W, the equations of motion and compatibility and applying Galerkin's method yields the following nondimensional modal equation:

$$\ddot{q}+\frac{E_2h^2}{\rho b^4}[\lambda_\infty^2 q+\beta_\infty q^3]=\frac{16}{9\rho h^2}p(t) \tag{7}$$

where

$$\beta_\infty=\beta_p^*+n\beta_c^* \tag{8}$$

$$\beta_c^*=\frac{\pi^4}{8E_2hr^4}\left(\frac{A_{22}^*-2A_{12}^*r^2+A_{11}^*r^4}{A_{11}^*A_{22}^*-A_{12}^{*2}}\right) \tag{9}$$

$$\beta_p^*=\frac{\pi^4}{9E_2h}\left[C_{10}+C_{01}+C_{11}+C_{02}+C_{20}+\frac{1}{2}(C_{21}+C_{12})\right] \tag{10}$$

$$\lambda_\infty^2=\frac{16\pi^4}{9E_2h^3r^4}[3D_{11}+2(D_{12}+2D_{66})r^2+3D_{22}r^4] \tag{11}$$

$$r=a/b$$

$$\begin{aligned}
C_{10}&=1/A_{22}^* & C_{12}&=K_3/(K_3^2-K_4^2)\\
C_{01}&=1/(r^4A_{11}^*) & C_{21}&=K_1/(K_1^2-K_2^2)\\
C_{11}&=2K_5/(K_5^2-K_6^2) & S_{11}&=-2K_6/(K_5^2-K_6^2)\\
C_{20}&=1/(16A_{22}^*) & S_{12}&=-K_4/(K_3^2-K_4^2)\\
C_{02}&=1/(16A_{11}^*r^4) & S_{21}&=-K_2/(K_1^2-K_2^2)
\end{aligned}$$

$$\begin{aligned}
K_1&=16A_{22}^*+4(2A_{12}^*+A_{66}^*)r^2+A_{11}^*r^4\\
K_2&=16A_{26}^*r+4A_{16}^*r^3\\
K_3&=A_{22}^*+4(2A_{12}^*+A_{66}^*)r^2+16A_{11}^*r^4\\
K_4&=4A_{26}^*r+16A_{16}^*r^3\\
K_5&=A_{22}^*+(2A_{12}^*+A_{66}^*)r^2+A_{11}^*r^4\\
K_6&=2A_{26}^*r+2A_{16}^*r^3
\end{aligned}$$

and $n=0$ for inplane movable boundary conditions and $n=1$ for inplane immovable boundary conditions.

The nonlinear static response is obtained by letting $\ddot{q}$ tend to zero and restricting $p(t)$ to remain constant. Thus, Eq. (7) becomes

$$\lambda_\infty^2 q+\beta_\infty q^3=\frac{16}{9}F_0 \tag{12}$$

where F_0 is the nondimensional force parameter

$$F_0=\frac{p_0b^4}{E_2h^4} \tag{13}$$

For the dynamic case when the forcing function, $p(t)$, is a harmonic function in the time, $\cos(\omega t)$, the solution of Eq. (7) can be obtained by using perturbation techniques. The frequency ratio for the nonlinear forced response can be shown[1-3]

$$\frac{\omega}{\omega_0}=\left(1+\frac{3}{4}\frac{\beta_\infty}{\lambda_\infty^2}q^2-\frac{16F_0}{9\lambda_\infty^2 q}\right)^{1/2} \tag{14}$$

By letting F_0 in Eq. (14) tend to zero, the frequency ratio for the free vibration is easily obtained.

It is known that damping has a significant effect on the response of structures. Therefore, the precise determination of the damping coefficient of a structure should be emphasized. The values of damping ratio $\zeta(=c/c_{cr})$ generally range from 0.005-0.05 for the common type of composite panel used in aircraft construction.[8-10] Once the damping ratio is determined from experiment or from existing data, it can be included in Eq. (7) and solved using the method of equivalent linearization, resulting in the following equation for the mean-square response

$$E[q^2]=\left(\frac{\lambda_\infty^4}{36\beta_\infty^2}+\frac{32S}{243\zeta\beta_\infty\lambda_\infty}\right)^{1/2}-\frac{\lambda_\infty^2}{6\beta_\infty} \tag{15}$$

where S is a nondimensional, forcing excitation, spectral density parameter defined as

$$S=\frac{S(f)}{\rho^2h^4(E_2h^2/\rho b^4)^{3/2}} \tag{16}$$

References

[1] Eisley, J. G., "Nonlinear Vibration of Beams and Rectangular Plates," Zeitschrift fur angewandte Mathematik und Mechanik, Vol. 15, 1964, pp. 167-175.

[2] Srinivasan, A. V., "Nonlinear Vibrations of Beams and Plates," *International Journal of Non-linear Mechanics*, Vol. 1, 1966, pp. 179-191.

[3] Chia, C. Y., "Large Deflection of Rectangular Orthotropic Plates," *Journal of Engineering Mechanics Division Proc. ASCE*, Vol. 98, No. EM5, 1972, pp. 1285-1298.

[4] Zaghloul, S. A. and Kennedy, J. B., "Nonlinear Behavior of Symmetrically Laminated Plates," *ASME Journal of Applied Mechanics*, Vol. 42, 1975, pp. 234-236.

[5] Iwinski, T. and Nowinski, J., "The Problems of Large Deflection of Orthotropic Plates (I)," *Archiwum Mechaniki Stosowanji*, Vol. 9, 1957, pp. 593-603.

[6] Caughey, T. K., "Nonlinear Theory of Random Vibrations—Advances in Applied Mechanics," Vol. II, C. S. Yih, (ed.), Academic Press, NY, 1971, pp. 209-253.

[7] Mei, C., "Response of Nonlinear Structural Panels Subjected to High Intensity Noise," AFWAL-TR-80-3018, WPAFB, March 1980.

[8] Wentz, K. R., Paul, D. B., and Mei, C., "Large Deflection Random Response of Symmetric Laminated Composite Plates," *Shock and Vibrations Bulletin*, Naval Research Laboratory, Washington, DC, May 1982.

[9] Caughey, T. K., "Equivalent Linearization Techniques," *JASA*, Vol. 35, 1963, pp. 1706-1711.

[10] Jacobson, M. J., "Advanced Composite Joints; Design and Acoustic Fatigue Characteristics," AFFDL-TR-71-126, WPAFB, April 1972.

[11] Jones, R. M., "Mechanics of Composite Materials," McGraw-Hill, New York, 1975.